

Some Insights into Stirling Machine Behavior

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ABSTRACT

Almost all practical cryocoolers embody, directly or indirectly, at least some elements of the Stirling cycle in its reversed form. Through its involvement in the analysis, design and development of both Stirling engines and refrigerating applications, the Stirling Cycle Research Group at the University of Canterbury has, over several years, encountered a number of aspects of the practical realisation of the Stirling cycle – and indeed some aspects of its ideal realisation – which are, in our view, easily overlooked or misinterpreted. Being unaware of these aspects may possibly lead to less-than-optimal design of Stirling machines and/or misinterpretation of modelling predictions and the performance data from actual machines.

The points of intended clarification are:

- The misleading, or at best easily misinterpreted, typical textbook representations of the ideal Stirling cycle in both p - v and T - s property diagrams;
- The difference between direct and reversed implementations of the Stirling cycle in terms of the role which the regenerator plays;
- A Carnot alternative to the regenerator: The regenerator is a device that ‘conditions’ the working gas locally and not ‘globally’ as would happen in a Carnot machine.
- Another difference between direct and reversed implementations of the Stirling cycle in terms of approaching the ideally isothermal heat transfer processes;

INTRODUCTION

While the Stirling cycle can be very simply explained in its idealized form, as can be found in any Stirling text book, the processes taking place in its practical realisation in various types of Stirling machinery deviate quite significantly from that ideal. Even more fundamentally, it is possible to misinterpret the implications that a simple explanation of the ideal cycle might suggest and carry such misinterpretations into the design process for real Stirling machines. Clearly a correct understanding of the complexity of any underlying thermodynamic and fluid-dynamic processes is essential for any successful Stirling machine design.

The Stirling Cycle Research Group at the University of Canterbury is a very small group of researchers, having its genesis in the initial development work on the DC version of the WhisperGen Micro Combined Heat and Power unit (MCHP). That postgraduate research was undertaken by Dr Don Clucas who now holds the position of Technical Manager in WhisperGen Ltd., the company which continues to develop the MCHP unit, which now is dominantly marketed in an AC version [WhisperGen¹]. In the years since that initial Stirling cycle work in the Dept. of Mechanical Engineering by Clucas², subsequent research activity has included

investigations by Weston³ into seal behavior and by Gschwendtner⁴ into regenerator characteristics. In more recent years there has been a shift away from power-producing Stirling machines (since the WhisperGen R&D team's effort is in this area) towards reversed Stirling cycle machines, first by Haywood⁵ looking at near-ambient temperature refrigeration possibilities and currently by Gschwendtner and Tucker⁶ on cryocooler design and development.

During this journey of understanding, development and design of Stirling-related machines the Group has encountered a number of aspects of the practical realisation of the Stirling cycle – and indeed some aspects of its ideal version – which are, in our view, easily overlooked or misinterpreted. At least one of these aspects has only become apparent because of the involvement in, and transition between, both direct and reversed versions of the cycle. To our knowledge these have not all been mentioned in the literature.

Drawing on the Group's combined experiences, and the outcomes of numerous discussions over cups of coffee and in front of whiteboards, this paper aims to highlight selected ideal and real-world behaviors of the Stirling cycle, thereby bringing to the attention of the Stirling community aspects which some may or may not be aware of.





LEGITIMACY OF P - V AND T - S STIRLING CYCLE REPRESENTATIONS

Virtually all introductory engineering thermodynamics textbooks include a description of the four processes which make up the ideal Stirling cycle, accompanied by a pair of property diagrams to illustrate the appearance of those processes. Thus, a typical textbook entry would be along the lines:

“The ideal Stirling cycle consists of four processes executed in sequence:

- 1-2 Reversible isothermal heat addition at T_H
- 2-3 Reversible isochoric heat transfer into storage (*externally adiabatic*)
- 3-4 Reversible isothermal heat rejection at T_L
- 4-1 Reversible isochoric heat transfer out of storage (*externally adiabatic*)

Under the assumption that the working fluid behaves as an ideal gas, the appearance of this cycle in pressure-volume (p - v) and temperature-entropy (T - s) diagrams is as shown in Figure 1 below.”

(As an aside, it might be noted that the versions of the property diagrams in Figure 1 are a little more informative than those typically presented in textbooks in that they convey visible indicators of the heat and work transfers that occur – both internally and externally – and the directions of those transfers. Arrows shown in the form  or  represent heat transfer, while the form  or  represents a work transfer. Such additional information is scarcely necessary for the audience of this present paper but these arrows are included because they are useful aids in some subsequent explanations.)

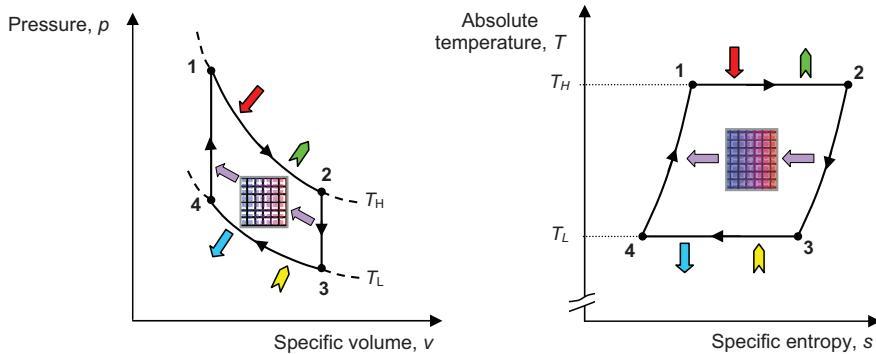


Figure 1. p - v and T - s diagrams for the ideal Stirling cycle.

Invariably, the accompanying explanation points out that, although the two isochoric processes may appear to imply that heat transfer is occurring during these constant volume processes (because of the fact that areas below reversible processes in T - s coordinates represent the heat transferred), from an *external* perspective these are in fact isentropic processes because those heat transfers are into and out of transient thermal energy storage in the form of an internal regenerator.

True though all of this may be, there is one important point of explanatory detail which invariably is omitted, and this can be a source of confusion in the minds of students who previously have been required to understand the circumstances under which the property values, and hence the thermodynamic state of a substance may be defined. (In fact it is often the case that such confusion will only arise for those students who think things through thoroughly, whereas acceptance of information at face value unfortunately is the much more common educational experience.)

The difficulty of representing the ideal Stirling cycle in this way relates primarily to the two isochoric processes (2-3 and 4-1 in Figure 1). On any thermodynamic property diagram, the representation of any state point – whether it be one of the “corner” states or an intermediate state on a process line – implies corresponding property values consistent with a state of thermodynamic equilibrium in the system or, at most, a very small perturbation from an equilibrium state. This requirement implies uniformity of property values at each point. This can be done with total legitimacy in representing other ideal cycles in either p - v or T - s coordinates. For example, in the Otto cycle (Figure 2(a)) the temperature can be assumed to be uniform throughout the gas volume at any instant; and in the Brayton/Joule cycle (Figure 2(b)) one is representing the changing state of a representative mass element of the gas as it moves sequentially through a series of steady state, linked components.

Thus, in these, and in other corresponding diagrams for most other ideal cycles, each state point around the cycle represents the true thermodynamic state of the substance at that point. In the case of the isochoric processes in the Stirling cycle, however, the representations of these processes are those of the changes of state of the working substance *on average* with, at each instant during the two isochoric processes, a very large range of temperature (and hence specific volume) existing throughout the mass of gas. Because of that non-uniformity of temperature, a state of equilibrium (or even near-equilibrium) does not exist in the gas and thus its thermodynamic state is undefined and hence, strictly, it cannot be represented on a property diagram.

If, for illustrative purposes, we neglect the dead volume of the gas in the regenerator and consider the heat transfer from the gas into the regenerator during the isochoric process 2-3 in Figure 1, at the commencement of the process the gas is uniformly at the high temperature T_H and State 2 is unambiguously defined. Likewise, at the conclusion of this process, the gas is uniformly at the low temperature T_L and State 3 is also unambiguously defined. In between, changing proportions of the gas are at these two extremes of temperature and the apparent “state” of the gas moves along the line 2-3 as these proportions change but it is erroneous to interpret this as the defined state of the gas at each point in succession.

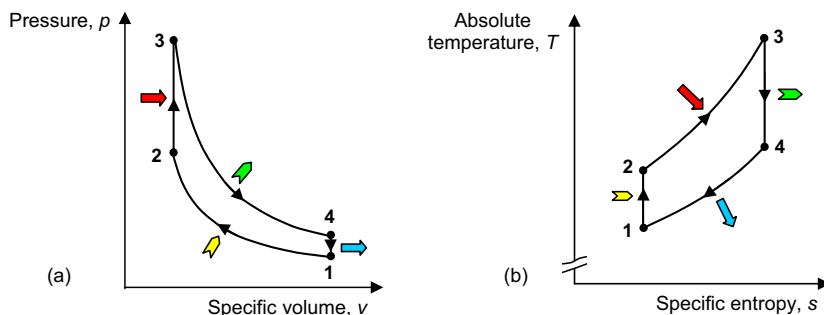


Figure 2. Other ideal cycle diagrams: (a) Otto p - v (b) Brayton/Joule T - s .

Certainly there will be at each instant a small elemental mass of the gas which happens to have pressure, temperature and specific volume values which are represented exactly by what the property diagrams suggest but that elemental mass will be neighboured by others which will share the same pressure (since that property is uniform at each instant in the ideal cycle) but different temperature and specific volume values. All that is able to be claimed with thermodynamic correctness and certainty is that the plot of pressure versus total volume portrays that relationship for what is experienced by the pistons which contain the gas. No such corresponding claim can be made, however, for the T - s representation of the ideal Stirling cycle. It is, in a sense, a 'hybrid cycle' which has elements of the simultaneously uniform gas state (exemplified in Otto cycle T - s representations) during the isothermal heat transfer processes. During the isochoric internal heat transfer processes, however, it resembles the 'run-around' nature of Brayton cycle T - s representation in that it is portraying – effectively in isolation – the changing state of an element of gas mass moving through the regenerator.

The potentially confusing consequences of these property diagram representations are most apparent in a T - s "property" diagram for the Vuilleumier cycle in which, effectively, a direct Stirling cycle drives a reversed Stirling cycle with a common and interconnected gas mass throughout the machine. The cyclical pressure fluctuations which the reversed cycle (1'-2'-3'-4' in Figure 3) normally would derive from a mechanically-driven compressor now are obtained, from a thermal compressor in which cyclical temperature fluctuations (1-2-3-4) occur within an appropriately controlled and varied containing volume [Wurm, Kinast, Roose and Staats⁷].

Figure 3, which effectively replicates Figure 3.11 in that reference, creates an impression of the changing state of the working fluid in an ideal Vuilleumier machine which almost certainly was not intended by the authors. It appears from this figure that the working fluid, in its entirety, follows a figure-of-eight sequence of processes.

The reality, even for the ideal machine, is quite different. For each of the four internal heat exchanges involving the two regenerators that are required in this cycle, the process lines 2-3, 4-1, 1'-2' and 3'-4' are not representing actual gas states because very non-uniform temperature conditions exist throughout the total mass of gas during those processes, as previously explained.

Additionally, Figure 3 conveys a possibly misleading impression of the timing sequence or the phase relationship between actions taking place in various components. For the ideal cycle being considered here, at any instant of time an isobaric condition will exist throughout the entire volume. While the superimposed isobars may be helpful in providing information about this equality of pressure between different states (1 and 1'; 2 and 2'; 3 and 3') at particular instants in the overall cyclic sequence, it may not be readily apparent that during a complete cycle the entire system pressure is oscillating, in phase, between the pressure extremes represented by the isobars 1-1' and 3-3'. On the other hand, the temperature – and consequently the specific volume of the gas – has very substantial variation throughout the gas mass at any instant, with the most extreme variation being along the isobar 2-2'. This particular instant in the cycle has been *preceded* by the isothermal heat input at T_H but is *immediately followed* by heat input at T_L .

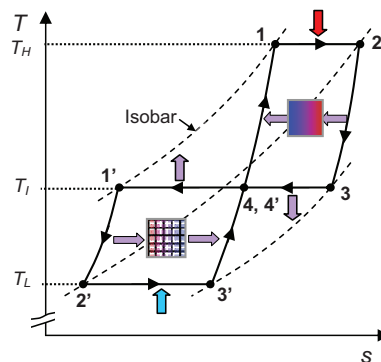


Figure 3. T - s representation of the ideal Vuilleumier cycle.

Unlike the much more familiar Stirling cycle, in the Vuilleumier cycle the total gas volume in the system remains constant throughout. This leads to the intriguing realisation that on a p - V plot (where V represents *total* volume) the entire cycle would appear as a single vertical line. Although uninformative, such a plot would at least have the virtue of being thermodynamically correct! On the other hand, a p - v (where v represents *specific* volume) would be meaningless because of the large range of specific volume values existing throughout the gas mass at any instant of time.

THE NECESSITY OF THE REGENERATOR

From a thermodynamic perspective it is a quite reasonable expectation that direct and reversed versions of the same thermodynamic cycle will have a thermal efficiency in direct mode which is the inverse of the COP as a heat pump in reversed mode (provided the cycle is fully reversible). Following on from this, one might reasonably expect that if an implementation of the cycle is able to operate in direct mode then it should be equally capable of operating in reversed mode.

Now consider the implementation of an engine operating on the Stirling cycle in direct (power producing) mode. Such an engine would include a regenerator to enable the two isochoric processes to be externally adiabatic, but it is still able to function – albeit with extremely low efficiency – without a regenerator being present. In this form of operation, necessarily there is large deviation from the ideal Stirling cycle but the key point is that it would operate and is able to produce a modest power output. The popular “coffee cup” or hand-warmed small Stirling engines demonstrate the truth of this statement.

Given the earlier statement, it would seem reasonable for a reversed version of this same device to generate a temperature difference, and hence act as a heat pump. In this reversed mode, however, there is one important difference which means that the expected temperature differential will not be achieved. This important difference is that in the case of the direct engine, the required temperature differential is *externally imposed*. On the other hand, for a reversed engine, the temperature differential can only be generated *as a consequence of the cyclic sequence of processes within the engine itself*. Obvious and inconsequential though this distinction may be, it is one which means that a direct Stirling engine is capable of operating without a regenerator whereas a reversed Stirling engine cannot.

Despite the previous section pointing out the shortcomings of T - s representations of the Stirling cycle we now use that same representation in explaining some aspects of the role of the regenerator. The justification in these present circumstances is that the temperature coordinate is a useful way of representing the “energy ladder” upon which thermal energy can be positioned. Mention of an energy “ladder” for thermal energy ties in with an analogy which is appropriate in describing the Second Law constraints under which a regenerator must operate. There is no difficulty in persuading joules of thermal energy to descend such a ladder, but enabling their ascent requires a “leg-up” in the form of a work input; otherwise we would have a device which contravenes the Clausius statement of the Second Law.

For a direct Stirling engine having access to existing thermal reservoirs maintained at temperature T_H and T_L , in the absence of a regenerator the T - S schematic of Figure 1 is amended to the form shown in Figure 5(a).

The isochoric processes 2-3 and 4-1 now achieve their associated heat transfers externally rather than internally via the regenerator. This has a doubly adverse consequence on the cycle's performance:

- The inward heat transfer during 4-1 represents additional external heat input for no additional gain in work output, so the efficiency of the cycle suffers inevitably;
- Both of the additional external heat transfers are non-isothermal, also moving the cycle performance away from that of the ideal Stirling cycle which, with the regenerator, matches the benchmark Carnot cycle in its efficiency. In other words, the ideal cycle is no longer both internally and externally reversible.

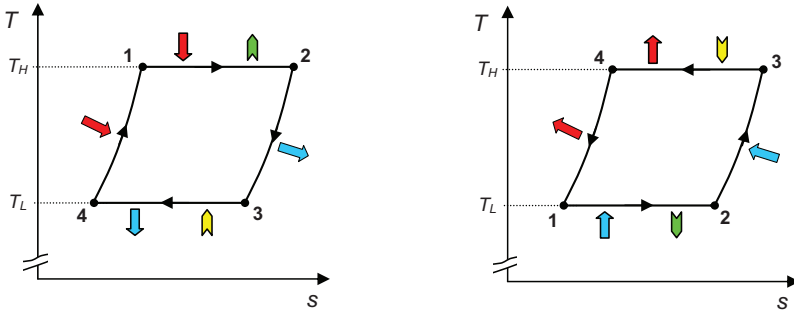


Figure 5. Representation of an ideal Stirling engine (a), and heat pump (b), both without a regenerator.

Despite this inevitable (and, for a practical Stirling engine, very severe) drop in performance, the engine nevertheless is able to operate without a regenerator. Why this should be possible is evident from Figure 5(a) where it can be seen that a thermal energy heat source at T_H also has the necessary favourable temperature differential to act as a heat source for not only the conventional isothermal heat addition process 1-2 but also throughout the now-additional isochoric heat addition process 4-1. Similarly, the thermal energy heat sink at T_L can also provide the necessary favourable temperature differential to act as a heat sink throughout the isochoric heat rejection process 2-3.

Now consider operation the same machine in reversed mode, i.e. as a heat pump. For the moment, let us assume that the desired overall temperature differential from T_L (heat input from a low temperature source) to T_H (heat output to a higher temperature sink) exists at the commencement of the machine's operation. In those circumstances we would be wanting the machine to operate and sustain the situation shown in Figure 5(b). Apart from a logical renumbering of the sequence of states to match the reversed direction, Figure 5(b) appears to be an exact reflection of Figure 5(a), leading to an understandable expectation that the required work input to this reversed cycle would bring about the desired heat transfer from T_L up to T_H .

Indeed, from that superficial perspective this machine appears to have avoided the trap of being overall a “Clausius Violator” but closer inspection of Figure 5(b) reveals that it is precisely that. Its fatal flaw lies in the external heat transfers required during the isochoric processes 2-3 and 4-1 because of the absence of the regenerator. Unlike the direct engine case described above, these required heat transfers are now in directions of “uphill” temperature gradient with respect to the heat source (now at T_L) and the heat sink (now at T_H). Because of the Second Law violation that these heat transfers would represent, the operation of this heat pump mode is impossible, so the initial assumption of the desired overall temperature differential ($T_H - T_L$) existing initially would result, in practice, in a decline back to an everywhere-isothermal condition with no enclosed area and therefore no net heat transfer.

With a regenerator present, this impossibility disappears because the required “downhill” temperature gradients to achieve the necessary heat transfers into and out of the gas exist locally within the regenerator during those same processes 2-3 and 4-1. Each of the required temperature levels exists as a residue of the internal energy storage and release processes taking place during the diagonally opposite process (4-1 and 2-3 respectively). In other words, these higher temperature mini-reservoirs within the regenerator are able to provide the necessary “leg up” to enable the required joules of energy to ascend the energy ladder. If this description makes it seem as if the work input is unnecessary, that is unintentional; the existence of the favourable temperature gradients within the regenerator of the reversed cycle exist only as an indirect consequence of the overall work input to the cycle.

The same arguments about the impossibility of the reversed Stirling cycle operating without a regenerator still hold when one removes the condition assumed above that the desired overall temperature differential ($T_H - T_L$) exists initially: in the regenerator-less form, the condition of no temperature differential initially would remain indefinitely. Some

clarification of this final point is called for, however, because the compression and expansion processes inherent in the Stirling cycle (direct or reversed) produce, as a direct consequence, temperature fluctuations as well. These are, however, cyclical in nature and not sustainable if there is no regenerator present. In a sense, then, a regenerator acts as a rectifier for these temperature fluctuations, effectively allowing thermal energy to move upwards (in temperature terms) without being cancelled out by a downwards movement half a cycle later.

This leads to a consideration of how a Stirling heat pump (*having* a regenerator) converges towards steady state heat pumping operation from a uniform temperature starting condition. Necessarily there is an incremental progression in which, during the first completed cycle, a very modest temperature profile exists within the regenerator matrix. This profile will still exist, and is added to, during the next completed cycle, and so on until, after a significant number of completed cycles, the final overall temperature distribution between T_L and T_H is established and cyclically maintained within the regenerator.

AN ALTERNATIVE TO THE REGENERATOR: A CARNOT CYCLE MACHINE

If one looks at a typical regenerator in Stirling machines, e.g. a fine steel mesh, one constantly wonders if the thermal benefit of putting such a highly obstructive object in the oscillating flow path of the working gas really outweighs the fluid-mechanical losses. However, this is universally acknowledged to be case.

These thoughts raise a question, however: Are there other possible ways of achieving the thermodynamic function of the regenerator in Stirling machines without incurring its significant fluid-mechanical losses? This, in turn, leads inevitably to the more fundamental consideration of what the regenerator actually does. Bridging the two temperature levels between which a Stirling machine operates, be it in the direct or in the reversed mode, the role of the regenerator is to increase the temperature of the gas by adding thermal energy from the regenerator matrix when the gas is transferring from the cold end. Conversely, when the gas transfers from the hot end its temperature drops by rejecting heat to the regenerator matrix.

Taking this consideration further, the just-posed question arises as to whether other means might be available to change the temperature of the working gas on its travel between the two temperature levels. We would wish, ideally, that this be achieved with no significant mechanical losses. The idea comes to mind that this change in temperature could be achieved by an appropriate change in the gas pressure, and this pressure change could be realized by a third piston which adiabatically compresses and expands the working gas during what conventionally are the isochoric processes in the Stirling cycle (Figure 6). With the two isochoric processes replaced by two varying volume adiabatic ones in addition to the still remaining two isothermal processes, the cycle has now been converted into a Carnot cycle.

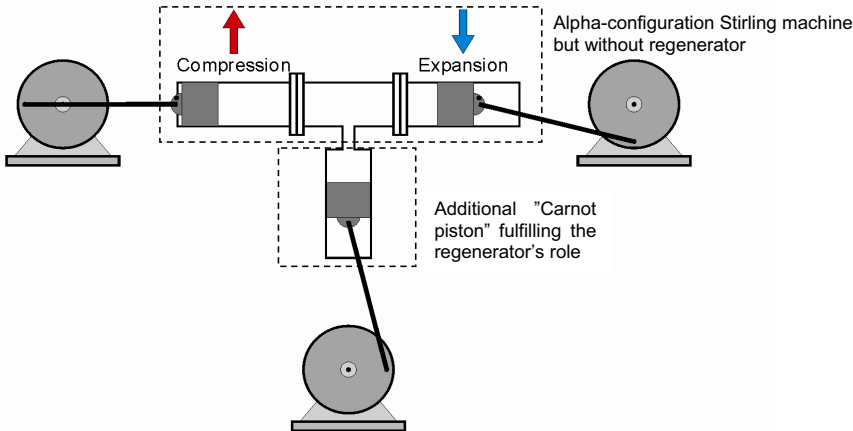


Figure 6. Carnot heat pump cycle realized by replacing the Stirling cycle regenerator with a third piston.

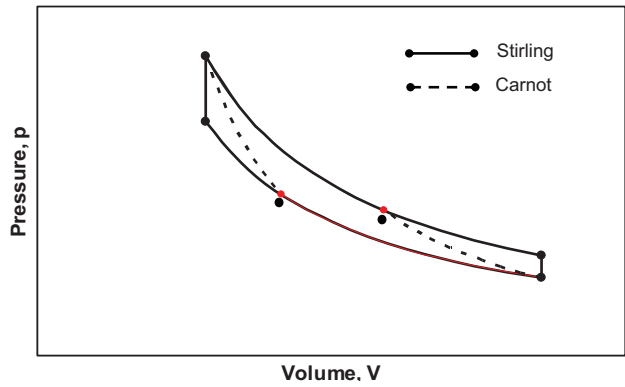


Figure 7. Comparison of the p - V loop between Stirling and Carnot cycles for given volumetric limits.

It is well known, however, that the Carnot cycle is inferior to the Stirling in terms of a smaller p - V loop and hence less usable work output in an engine having given volumetric limits (Figure 7). However, the difference between the Stirling and the Carnot cycle becomes less severe for smaller temperature differences as is the case in heat pumps, for example.

To illustrate this point, Figure 8 depicts the ratio of rejected heat in an ideal Carnot heat pump to that for an ideal Stirling heat pump for a fixed cold end temperature of 260 K. This ratio is plotted over a range of heat rejection temperatures and for various volumetric ratios V_{max}/V_{min} . It can be seen that the smaller the difference between the heat absorption and heat rejection temperature becomes, the smaller the difference is between a Carnot and a Stirling heat pump. This is even more so, the larger the volumetric ratio V_{max}/V_{min} becomes. For example, for a heat rejection temperature of $T_H=320$ K and a volumetric ratio of $V_{max}/V_{min}=4$, the rejected heat in an ideal Carnot heat pump is almost 80% of that in an ideal Stirling heat pump under similar conditions. This does not sound too discouraging.

The choice of a fixed cold end temperature of 260 K may appear to be atypical for Stirling cycle applications which would be more likely to be substantially lower than this (for cryogenic applications) or else, for a direct engine, a little higher than this and working in conjunction with a high temperature source at perhaps 1000 K. When plots similar to Figure 8 are produced for either of these conditions the Carnot compares much less favourably with the Stirling, leading to the conclusion that if this concept is to be pursued further, its usefulness would be restricted to near-ambient refrigeration applications.

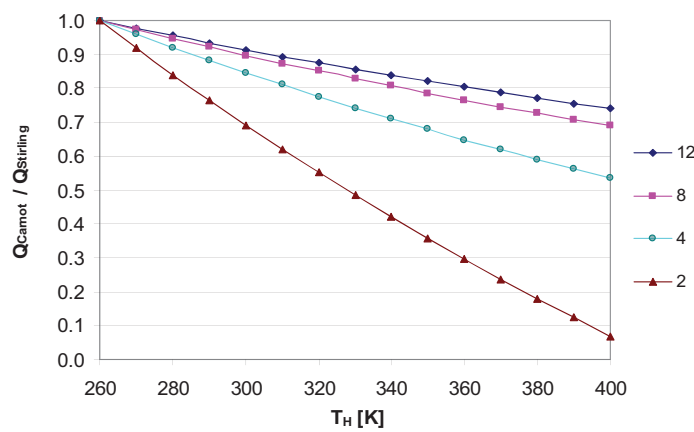


Figure 8. The ratio of rejected heat in an ideal Carnot heat pump to that for an ideal Stirling heat pump for a fixed cold end temperature of 260 K, plotted over a range of heat rejection temperatures, T_H , and for volumetric ratios V_{max}/V_{min} ranging from 2 up to 12.

Within this limitation, given the not too dissimilar performance between the two cycles under assumed ideal conditions, one wonders if the lack of an obstructive component in a real machine, as represented by the regenerator in a Stirling machine, may allow the Carnot cycle to overcome this theoretical shortcoming and actually perform better than the Stirling cycle.

Encouraging though this speculative suggestion may appear to be, it is acknowledged that that pressure changes caused by the third piston occur everywhere in the system and may have a detrimental effect on the direction of heat transfer in the heat exchangers. In other words, the added pressure fluctuations may shift the average gas temperatures in each of the heat exchangers to less favourable values. Should this be the case it could be argued that the effect of “conditioning” the working gas by localized heat transfer in the regenerator is still superior to a “Carnot piston” despite the higher mechanical losses.

While this is not more than academic speculation, a more detailed simulation model (by using Sage, for example) might shed more light on this question.

THE ISOTHERMAL-TO-ADIABATIC SPECTRUM FOR THE HEAT TRANSFER PROCESSES IN REAL STIRLING MACHINES

As well as the already-discussed role of the regenerator, there is another fundamental constraint on any real Stirling machine which has different consequences in the implementation of the Stirling cycle in direct mode as compared with reversed (or heat pump/refrigerator) mode.

The starting point for this particular discussion is the statement in many Stirling textbooks that the ideally isothermal expansion and compression processes are, in reality, closer to adiabatic than they are to isothermal processes. In polytropic terms, this statement implies that the effective polytropic index of the process tends to be closer to the adiabatic value (1.40 for air, behaving ideally at 300 K; 1.66 for helium) than it is to the isothermal value of unity.

Upon closer scrutiny, however, there appears to be a significant difference between Stirling machines operating in direct or reversed mode with respect to what determines the polytropic index. Thermodynamically speaking, it is the amount of thermal energy that is being exchanged between the system, i.e. the working gas, and the surroundings that determines how close to the adiabatic or isothermal limit the process is. If no heat is exchanged with the surroundings, the process becomes adiabatic. With no change in internal energy of the working gas and, consequently, total conversion of heat to work (or vice versa) the process would be isothermal.

For a heat pump this scenario makes perfect sense. If, for example, the working gas is being compressed by a piston, the piston work is absorbed by the working gas and increases its internal energy. With a temperature gradient now existing between the gas and the surroundings, heat is rejected through the heat exchanger. However, the amount of thermal energy transferred depends on the typical limitations of heat transfer processes, namely the available time and the thermal resistance through the heat exchanger walls. Were there no such limitations, the complete amount of work would be transferred to the surroundings in the form of heat with no change of internal energy of the gas, making the process isothermal. At the other end of the spectrum with infinite limitations prohibiting any heat transfer, the process would be adiabatic. Obviously, similar reasoning is true for the expansion process in a heat pump or refrigerator.

Those expecting something similar in heat engines will be surprised to find a different mechanism governing the degree to which the expansion or compression process is polytropic. Let's take a look at the expansion process in a heat engine. The working gas absorbs thermal energy from the heat source and expends work on the piston. How close to isothermal or adiabatic is this process? Do the limitations of heat transfer again determine the polytropic index? The surprising answer is no. If we consider cause and effect or, more appropriately, what is the driving force behind the process and what is the driven part, we realize that there is a different mechanism at work in direct Stirling machines compared to those in reversed mode.

While in the case of a heat pump/refrigerator the heat transfer is the driven part caused by work expended on the gas by the piston, in a heat engine it is the heat transfer that is the driving force behind the expansion and compression (or: contraction) process. In these circumstances it

is the characteristics of the load that determines how close to isothermal or adiabatic the process in a heat engine is, rather than the heat transfer process being the determining factor. In other words, the 'quality' of the heat exchanger has no influence whatsoever on the polytropic index of the expansion and compression process in a heat engine. This rather surprising insight becomes more apparent if one realizes that the gas only gets what it 'sees': however small or large the amount of heat is that it absorbs or rejects, this is the cause for any piston motion in a heat engine. Depending on how quickly or slowly the piston can be moved against its applied external load, the temperature of the working gas will deviate from the temperature of the heat source or sink and this alone will determine the polytropic index of the expansion or compression process.

As a final interesting note, in a heat engine the polytropic index of the expansion and compression process is not limited to the range from isothermal to adiabatic. In fact, the process could even be isobaric or nearly isochoric if the external load on the piston is too high.

CONCLUSIONS

Greater care has to be exercised in interpreting the representations of pressure-volume and temperature-entropy relationships for the Stirling cycle (and its derivatives) than is required for the corresponding plots of other thermodynamic cycles.

Conclusions drawn from considering the operation of the direct Stirling cycle do not automatically translate to the reversed cycle version, even allowing for the apparent inverse relationship of the cycles. One such asymmetry becomes apparent when the possibility of operation without a regenerator is considered.

An appropriately phased additional piston has been postulated as a possible alternative to a regenerator to achieve a similar change in the gas temperature, thereby potentially avoiding the fluid-mechanical losses incurred through a regenerator. This approach results in a Carnot machine but the apparent potential is constrained by considerations of the achievable volume ratio. Furthermore, the associated global gas pressure changes may have undesirable influences on the effectiveness of the two external heat exchange processes.

A second direct/reversed Stirling cycle asymmetry has been identified, this one pertaining to how closely the ideally isothermal external heat transfers may be approached in practice.

These topics illustrate that, despite the outward simplicity of the Stirling cycle, there are hidden complexities which can stand in the way of gaining a full understanding of the often inter-dependent processes taking place. It seems doubtful that Reverend Robert Stirling was distracted by such confounding thought processes almost 200 years ago when he devised the cycle that bears his name.

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